

## A Wideband Microwave Solid-State FM Deviator

Existing frequency modulated microwave sources are typically limited to either relatively narrow frequency deviations ( $<\pm 20$  MHz) or relatively low baseband response ( $<10$  MHz), unless complicated techniques are employed. The objective of the work described in this correspondence was to develop a simple solid-state microwave deviator with greatly extended deviation and baseband response. Specifically, a frequency deviation of

$\pm 100$  MHz centered at 1.3 GHz with a linearity of less than 2 MHz, a power output of 120 mW flat to 1 dB and a baseband response from dc to 60 MHz were established as goals.

The FM deviator designed for this purpose utilized a transistor oscillator which operates at its fundamental frequency. In order to obtain the required broadband deviation characteristics, a simple LC resonant circuit containing the baseband-voltage dependent collector-to-base capacitance of the transistor, a varactor and an external induc-

tor was utilized, as shown in Fig. 1. In this way the ratio of energy stored in the variable capacitances to the total energy stored in the resonant circuit was maximized. Furthermore, since the higher voltage is applied to the transistor as the frequency increases, the output power is maintained relatively flat over the whole range of interest.

Linearization of the frequency-voltage characteristics was accomplished by shaping the input voltage characteristics. For this purpose Schottky barrier diodes were used, as shown in Fig. 1.

Using Fairchild MT1038 transistors operating at L-band, several FM deviators were constructed. The best of these had a linearity of less than 2 MHz for a peak-to-peak deviation of 200 MHz and a differential nonlinearity of less than  $\pm 6$  percent. The deviators are capable of providing an output power of greater than 100 mW over the band, flat to within 1 dB. Figure 2 shows the RF output response and the frequency versus baseband voltage characteristic. The baseband frequency characteristic is flat to within  $\pm 1$  dB from dc to 60 MHz as shown in Fig. 3. On a spectrum analyzer with 1 kHz IF bandwidth the noise level of the deviator was more than 60 dB below the carrier at 100 kHz away from the carrier. No spurious responses were observed over a several hundred megahertz range.

Similar results have also been obtained with push-pull oscillators where the varactor and RFC's are completely eliminated.

W. J. CLEMETSON  
Bell Telephone Labs., Inc.  
Murray Hill, N. J.

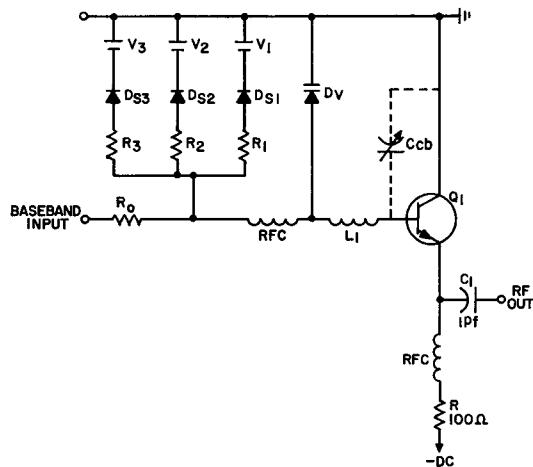


Fig. 1. Schematic diagram of linearized L-band FM deviator.

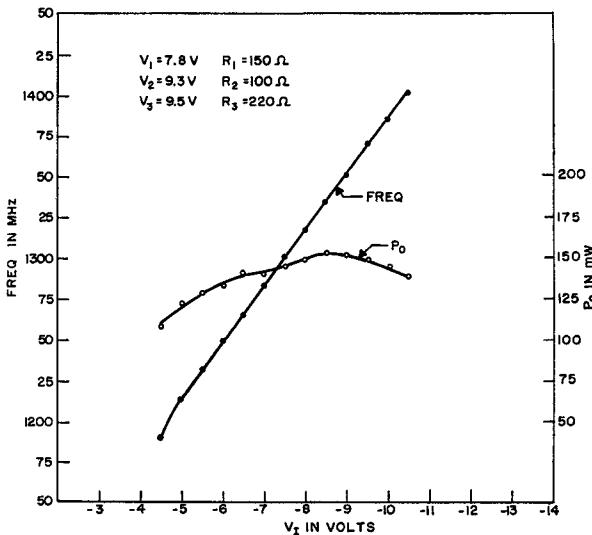


Fig. 2. RF characteristics of FM deviator.

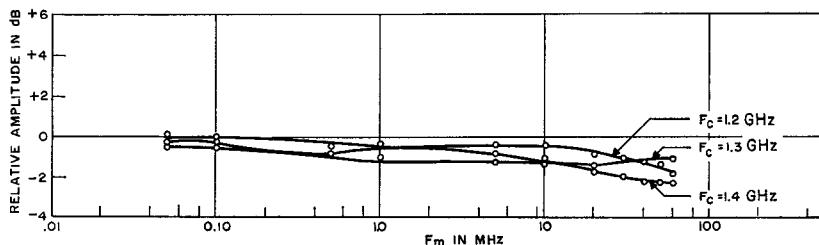


Fig. 3. Baseband response for three RF carrier frequencies.

Manuscript received November 23, 1966.

## Comments on "Longitudinal Waves in a Hot-Nonuniform Plasma"

In a recent correspondence Lonngren<sup>1</sup> derived the general wave equation for an inhomogeneous isotropic warm electron plasma in the form:

$$-\nabla \times \nabla \times \vec{E} + \frac{a^2}{c^2} \nabla \nabla \cdot \vec{E} + k_0^2(1-X)\vec{E} = \frac{a^2}{c^2} \frac{\nabla N_0}{N_0} \nabla \cdot \vec{E} \quad (1)$$

where  $a = [\gamma K T_0 / m]^{1/2}$  = acoustic velocity of the electron gas,  $X(\vec{r}) = e^2 N_0(\vec{r}) / \omega^2 m$ ,  $k_0 = \omega / c$ ,  $k_1 = \omega / a$  and  $\gamma = C_p / C_v$ .

The general wave equation for an inhomogeneous gyrotrropic warm plasma with a static magnetic field was derived by Unz<sup>2</sup> in the form:

Manuscript received November 28, 1966. The research reported in this work was supported in part by the National Science Foundation.

<sup>1</sup> K. E. Lonngren, "On longitudinal waves in a hot-nonuniform plasma," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-14, pp. 494-495, October 1966.

<sup>2</sup> H. Unz, "Wave propagation in inhomogeneous gyrotrropic warm plasmas," *Am. J. Phys.* (to be published).

$$\begin{aligned}
& -\frac{1}{k_0^2} \nabla \times \nabla \times \bar{E} + \frac{1}{k_1^2 U} \nabla \nabla \cdot \bar{E} \\
& + \left(1 - \frac{X}{U}\right) \bar{E} + \frac{i \bar{V}}{U} \\
& \times \left( \bar{E} - \frac{1}{k_0^2} \nabla \times \nabla \times \bar{E} \right) = \frac{1}{k_1^2 U} \\
& \cdot \left( \frac{1}{\gamma} \frac{\nabla N_0}{N_0} - \frac{\gamma - 1}{\gamma} \frac{\nabla T_0}{T_0} \right) (\nabla \cdot \bar{E}) \\
& + \frac{1}{U} \nabla \left\{ \frac{1}{k_1^2} \left( \frac{\gamma - 1}{\gamma} \frac{\nabla N_0}{N_0} \right. \right. \\
& \left. \left. - \frac{1}{\gamma} \frac{\nabla T_0}{T_0} \right) \right. \\
& \left. \cdot \left( \bar{E} - \frac{1}{k_0^2} \nabla \times \nabla \times \bar{E} \right) \right\} \quad (2)
\end{aligned}$$

where  $\bar{V}(\bar{r}) = (e\mu/\omega m) \bar{H}_0(\bar{r})$ ,  $\bar{H}_0(\bar{r})$  being the inhomogeneous magnetostatic field, and  $U(\bar{r}) = 1 - iZ(\bar{r}) = 1 - i\nu(\bar{r})/\omega$ ,  $\nu(\bar{r})$  being the average collision frequency, and the plasma is inhomogeneous both in average density  $N_0(\bar{r})$  and in average temperature  $T_0(\bar{r})$ .

For the particular case under consideration by Lonngren<sup>1</sup> one has  $U=1$ ,  $\bar{V}=0$ ,  $\nabla T_0=0$ , ( $T_0=\text{constant}$ ), and  $k_1=\omega/a=\text{constant}$ , and (2) becomes:

$$\begin{aligned}
& -\nabla \times \nabla \times \bar{E} + \frac{a^2}{c^2} \nabla \nabla \cdot \bar{E} \\
& + k_0^2 (1 - X) \bar{E} \\
& = \frac{a^2}{\gamma r^2} \frac{\nabla N_0}{N_0} \nabla \cdot \bar{E} + \frac{a^2}{c^2} \frac{\gamma - 1}{\gamma} \\
& \Delta \left\{ \frac{\nabla N_0}{N_0} \cdot \left[ \bar{E} - \frac{1}{k_0^2} \nabla \times \nabla \times \bar{E} \right] \right\}. \quad (3)
\end{aligned}$$

For small inhomogeneities  $|\nabla N_0/N_0| \ll 1$ , (1) and (3) are identical, but they are not identical in general.

The reason for the disagreement between (1) and (3) seems to be the suggestion by Lonngren<sup>1</sup> to use the equation of state

$$p_1 = \gamma K T_0 N_1 \quad (4)$$

for the inhomogeneous plasma. The equation of state for the adiabatic process and no viscous effects has been given by Unz<sup>3</sup> in the form:

$$\frac{Dp}{Dt} = \gamma K T \frac{DN}{Dt}. \quad (5a)$$

Using small signal theory in (5a) for inhomogeneous nondrifting plasma, it has been pointed out by Rao and Unz<sup>4</sup> that the adiabatic equation of state becomes for the present case:

$$\begin{aligned}
& \frac{\partial p_1}{\partial t} + \bar{u}_1 \cdot \nabla p_0 \\
& = \gamma K T_0 \left[ \frac{\partial N_1}{\partial t} + \bar{u}_1 \cdot \nabla N_0 \right]. \quad (5b)
\end{aligned}$$

Only for the particular case of homogeneous plasma  $\nabla N_0 = \nabla p_0 = 0$  (5b) agrees with (4). However, for the inhomogeneous plasma one

should use the equation of state (5b) rather than (4) as suggested by Lonngren,<sup>1</sup> and as a result one will obtain the wave equation (3) for the inhomogeneous warm plasma given by Unz<sup>2</sup> rather than (1) given by Lonngren.<sup>1</sup>

D. KALLURI

H. UNZ

Dept. of Elec. Engrg.  
University of Kansas  
Lawrence, Kan. 66044

#### Author's Reply<sup>5</sup>

Kalluri and Unz note that at the low-frequency limit of a collision dominated plasma, the pressure obeys an equation of state of the form<sup>6</sup>

$$\frac{d}{dt} (P n - r) = 0. \quad (1)$$

This modifies our equation (12)<sup>1</sup> to

$$\begin{aligned}
E_z = C_2 \epsilon^{-[(\gamma-2)/\gamma] [\dot{N}_0/N_0] z} \cos \left\{ \frac{k_0}{\alpha} \left[ 1 - \frac{\omega^2 p}{2\omega^2} \right] z \right. \\
\left. + \frac{\alpha}{k_0} \frac{\gamma - 1}{2\gamma} \left[ \frac{\dot{N}_0}{N_0} - \left( \frac{\dot{N}_0}{N_0} \right)^2 \right] z + C_1 \right\}. \quad (2)
\end{aligned}$$

#### ACKNOWLEDGMENT

We wish to thank D. Kalluri and H. Unz for this comment and Profs. H. C. S. Hsuan and D. C. Montgomery for discussions concerning it.

K. E. LONNGREN  
Dept. of Elec. Engrg.  
University of Iowa  
Iowa City, Iowa

<sup>5</sup> Manuscript received January 3, 1967.

<sup>6</sup> D. C. Montgomery and D. A. Tidman, *Plasma Kinetic Theory*. New York: McGraw-Hill, 1964. pp. 194-209.

solution has been obtained using the slightly simplified cross section of Fig. 1(b), in which the outer conductor slots are neglected. In addition, the validity of this approximate solution has been demonstrated experimentally.

#### THEORETICAL RESULTS

The effective dielectric constant  $\bar{\epsilon}$  and the characteristic impedance  $Z_0$  of the configuration of Fig. 1(b) are obtained from its static, unit-length interconductor capacitance,  $C(pF/in)$ , using the fundamental relationships<sup>8</sup>

$$\begin{aligned}
\bar{\epsilon} &= \frac{C}{C_0}, \quad Z_0 = \frac{84.9}{C_0(pF/in)\sqrt{\bar{\epsilon}}}, \\
C_0 &= C |_{\bar{\epsilon}=1}. \quad (1)
\end{aligned}$$

To calculate  $C$  in this case in which its direct solution for the given cross section does not exist, we locate the latter in the complex  $\phi$  plane [Fig. 1(b)], and use the property that  $C$  is invariant under conformal transformations  $t=t(\phi)$  of the  $\phi$  plane.<sup>3</sup> We then select  $t$  to yield that transformed cross section for which  $C$  can most conveniently be calculated directly. In this case, we choose the familiar bilinear transformation

$$t = \frac{1 + \phi}{1 - \phi} = u + jv; \quad \phi = \mu + j\psi \quad (2)$$

which yields the transformed  $t$ -plane cross section with new dimensions, as shown in Fig. 1(c).  $C$  cannot be calculated directly in the  $t$ -plane, nor is there any convenient conformal transformation of the  $t$ -plane to yield a configuration amenable to exact calculation of  $C$ . However, if we approximate the  $t$ -plane cross section by the  $t'$ -plane cross section of Fig. 1(d) (valid particularly for small  $\arg t$  and hence, for small substrate thickness  $2\Delta$ , and exact in the limit  $\Delta=0$  or  $\epsilon=1$ ) and use the method of images,<sup>3</sup> the resulting configuration can be solved directly for  $\bar{C} \approx C$ . In specific, we define  $C'$  as the coupled-strip capacity between the transformed center conductor and its image. Then the desired  $C$ , approximated by  $\bar{C}$ , the capacity between the center conductor and the imaginary-axis ground conductor, is given by<sup>4</sup>

$$\begin{aligned}
C \approx \bar{C} = 2C' = 0.45 \left[ \frac{K(k)}{K(k')} \right. \\
\left. + \frac{1}{2} (\epsilon - 1) \frac{K(k_1)}{K(k_1')} \right] pF/in \quad (3)
\end{aligned}$$

where

$$\begin{aligned}
K(\alpha) &= \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \alpha^2 \sin^2 \theta}} \\
k' &= \sqrt{1 - k^2} = \frac{s}{2a + s} \\
k_1' &= \sqrt{1 - k_1^2} = \frac{\tanh \left( \frac{\pi}{4} \frac{s}{\tau} \right)}{\tanh \left( \frac{\pi}{4} \frac{2a + s}{\tau} \right)}.
\end{aligned}$$

<sup>3</sup> R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 119-169.

<sup>4</sup> R. F. Frazita, "Transmission line properties of coplanar parallel strips on a dielectric sheet," M.S. thesis, Polytechnic Institute of Brooklyn, New York, 1965.

<sup>1</sup> H. Unz, "Wave propagation in drifting isotropic warm plasma," *Radio Sci.*, vol. 1, pp. 325-328, appendix A, March 1966.

<sup>2</sup> S. S. Rao and H. Unz, "On the adiabatic equation of state for inhomogeneous warm plasmas," *Proc. IEEE (Letters)*, vol. 54, pp. 1224-1225, September 1966.

<sup>3</sup> H. C. Okean, "Integrated microwave tunnel diode devices," 1966 *G-MTT Symp. Digest*, pp. 135-140, May 1966.

<sup>4</sup> H. C. Okean, "Microwave amplifiers employing integrated tunnel diode devices," to be published.